7-4 Remainder and Factor Theorems

Solve.
$x^3 + 4x^2 - 15x - 18 = 0$
If x - 3 is a factor.

3 | 1 4 -15 -18
3 21 18
1 7 6 : 0
$(x-3)(x^2+7x+6) = 0$
$(x-3)(x+6)(x+1) = 0$
$\{3, -6, -1\}$

Solve.
$x^3 + 7x^2 + 2x - 40 = 0$
If x - 2 is a factor.

2 | 1 7 2 -40
2 18 40
1 9 20 : 0
$(x-2)(x^2+9x+20) = 0$
$(x+5)(x+4) = 0$
$\{2, -5, -4\}$

Solve.
$x^3 - 2x^2 + 9x - 18 = 0$
If x - 2 is a factor.

2 | 1 -2 9 -18
2 0 18
1 0 9 : 0
$(x-2)(x^2+9) = 0$
$x = 2$ $\sqrt{x^2} = \sqrt{-9}$
$\{2, \pm 3i\}$ $x = \pm 3i$

When it is a factor, what can you say about the remainder?
$= 0$

Is it a factor?

$f(-2)$
$f(x) = x^3 + x^2 + 3x + 3$
Is x + 2 a factor? No

-2 | 1 1 3 3
-2 2 -10
1 -1 5 : -7

$f(x) = x^3 + x^2 + 3x + 3$
Find f(-2).

$= (-2)^3 + (-2)^2 + 3(-2) + 3$
$-8 + 4 - 6 + 3$
$f(-2) = -7$

$f(x) = x^3 + x^2 + 3x + 3$
Is x + 3 a factor?

$-3 \mid 1 \;\; 1 \;\; 3 \;\; 3$
$\quad\quad -3 \;\; 6 \;\; -27$
$1 \;\; -2 \;\; 9 \;\vdots\; -24$
No

$f(x) = x^3 + x^2 + 3x + 3$
Is x + 1 a factor? yes

$-1 \mid 1 \;\; 1 \;\; 3 \;\; 3$
$\quad\quad -1 \;\; 0 \;\; -3$
$1 \;\; 0 \;\; 3 \;\vdots\; 0$

Find k such that
$2x^4 + x^3 + 5x^2 - 6x + k \div x + 2$ has a remainder of 5.

$-2 \mid 2 \;\; 1 \;\; 5 \;\; -6 \;\; k$
$\quad\quad -4 \;\; 6 \;\; -22 \;\; 56$
$2 \;\; -3 \;\; 11 \;\; -28 \;\vdots\; k+56$

$k + 56 = 5$
$k = -51$

$2x^4 + x^3 + 5x^2 - 6x + k$
Find k such that x + 2 is a factor.

$k + 56 = 0$
$k = -56$

Remainder Theorem (summary)
The remainder of $f(x) \div (x - a)$ is f(a).

$a \mid$ ⋮ -7 $\quad f(a) = -7$

Factor Theorem (summary)
The binomial (x - a) is a factor of f(x) iff f(a) = 0.

$f(x) = 3x^4 - 2x^3 + x^2 - 2$

Use synethic subst.

f(4) = 654

4 | 3 -2 1 0 -2

12 40 164 656

3 10 41 164 | 654

f(2) = 34